braic means, others (notably Huygens and de Sluse) provided purely geometric arguments that showed the failure of his constructions. Thus, we need not think that Hobbes was somehow working out a nonalgebraic alternative to traditional mathematics, which alternative must be judged according to some sui generis standard.

Another problem for sociological positivism in this case concerns the remarkable fact that the sociological analysis of Hobbesian mathematics must, it seems, take a line diametrically opposed to the sociological reconstruction of Hobbes's dispute with Boyle. Shapin and Schaffer imagine that Hobbes's insistence that natural philosophy proceed by a priori deductions from mechanical first principles is a causal consequence of his political absolutism. Just as the sovereign must lay down absolute and unchallengeable dictates to govern the commonwealth, so the properly developed natural philosophy must start with abstract hypotheses rather than messy empirical data.[6] Boyle, in contrast, seeks a "commonwealth" or "balance of powers" guaranteed by publicly certified agreements, and his insistence upon the role of experiment is supposed to be linked to this political agenda. But in the realm of mathematics, it is Hobbes who appeals to the empirical adequacy of his results while his opponents insist upon the unchallenged (and unchallengable) axioms of traditional geometry. Hobbes claimed, for example, that two lines in a construction "are equal, at least so closely that the difference cannot be discovered either by the senses or by reasoning" (*CTH* 2:609). The rest of the mathematical world greeted such appeals to approximation with derision, arguing that the issue is not whether two lines have any discernible difference in length, but whether their equality has been demonstrated on the basis of axiomatic first principles. Further, Wallis attacked Hobbes's geometric principles for importing questionable physical principles into the realm of mathematics and thereby failing to respect the abstract, universal, and unchallengable character of geometric reasoning.

6. Shapin and Schaffer insist that "[t]he force by which Leviathan lays down and executes the laws of the commonwealth is therefore the same force that lies behind geometrical inferences" (1985, 153). Later they claim that Hobbes's dispute with Boyle "was a contest about power and assent. Geometry was normative for social relations because it was consistent with the Hobbesian model of assent" (1985, 328). Barnes, Bloor, and Henry agree that "[t]he model of knowledge for Hobbes was geometrical reasoning, which could secure total and irrevocable agreement. . . . Hobbes's method in natural philosophy, as in his political theory, ultimately depended upon unquestioning obedience to an absolute authority" (1996, 153). It may be useful to point out that by the time of *PRG,* Hobbes seems to have given up on the idea that his (or any other) geometrical principles could garner universal assent.

Ought we therefore to conclude Hobbes was a closet ally of the broad popular masses while his opponents in the Royal Society were crypto-absolutists, whose political agenda can be glimpsed in their unswerving allegiance to the authority of Archimedes?

A further reason for skepticism about a purely sociological approach to this controversy is the fact that Wallis was involved in bitter disputes throughout his career, and there is no way to construct an analysis of these battles that could reduce them to some unifying set of social factors. Richard S. Westfall described Wallis as a "bellicose character engaged in endless quarrels and controversies" (1958, 18), and this opinion can be upheld by a even the most cursory look at his published works. The catalog of Wallis's controversial writings begins with *Truth Tried,* a 1642 critique of Lord Robert Brooke's treatise *The Nature of Truth*. It continues with the *Commercium Epistolicum* of 1658, which is a record of the exchange of letters between the French mathematicians Fermat and Bernard Frenicle de Bessey on the one side and Wallis and Brouncker on the other. This exchange began when Fermat posed some number-theoretic challenge problems to the English mathematical community through the mediation of Thomas White and Kenelm Digby; in the course of time, however, Wallis and Fermat traded hostile comments on the relative merits of Wallis's method of inductions as well as priority claims for French and English mathematicians in the discovery of important results.[7] Wallis pursued his hostility toward French mathematicians in his *Treatise of Algebra* of 1685, which contained a fanciful history of algebra that attributed essentially all of the significant developments in that subject to English authors, and particularly to Thomas Harriot.[8] Another of Wallis's mathematical polemics was his 1657 *Adversus Meibomii, de Proportionibus Dialogus*—a piece whose level of vituperation led Barrow (who held no high opinion of Meibom's efforts) to call it a "diatribe" (*LM* 18, 293).

Mathematical matters were not the only source of Wallis's contributions to the literature of controversy. Theological issues, such as the

7. This dispute is examined in Mahoney 1994, 335–47. It is also summarized in Scott 1938, chap. 5.

8. In the preface to the *Treatise of Algebra,* he declares that Harriot "hath taught (in a manner) all that which hath since passed for the *Cartesian* method of *Algebra;* there being scarce any thing of (pure) *Algebra* in *Des Cartes,* which was not before in *Harriot;* from whom *Des Cartes* seems to have taken what he hath (that is purely *Algebra*) but without naming him" (*Treatise of Algebra* preface, sig. a2). French mathematicians did not find this amusing.

nature of the Trinity, the propriety of infant baptism, or the proper account of the Christian Sabbath provided him with the chance to engage in public controversy with a variety of authors that produced no fewer than eight published letters on the Trinity and numerous other minor pieces of polemical theology (Wallis 1692a, 1692b, 1696, 1697). Furthermore, Wallis became embroiled in a bitter dispute with William Holder concerning the credit for teaching a deaf-mute to pronounce some words of English. The result was his *Defence of the Royal Society* (Wallis 1678), which sharply rebuked Holder for the accusations in his *Supplement to the Philosophical Transactions of July 1670* (Holder 1678). Nor were Wallis's quarrels restricted to his published works; his letters clearly show him to have been a man eager for a fight. Scott reports that "Wallis was of a highly contentious disposition. His correspondence, unhappily, leaves no room whatever for doubt on that point. No man ever scorned personal popularity more completely than he" (1938, 88). It is far from credible that all of these quarrels could be correlated with some social or political interests, and the natural interpretation of Wallis's penchant for controversy must be in terms of individual psychological factors rather than some fanciful sociological just-so story in which he appears as the defender of a form of life. The relevance of this to the dispute with Hobbes should be obvious, as it shows that we need not seek for some underlying set of social interests to explain why Wallis pursued his battle with Hobbes so vigorously.

A final reason for treating the social factors as secondary in the dispute between Hobbes and Wallis is that they are not a constant. We saw in chapter 2 that the original impetus to quarrel involved the perceived threat that Hobbes posed to the universities. Yet the question of the status of English universities was settled well before the Restoration and disappeared from the exchanges between Wallis and Hobbes, while mathematical questions remained at center stage for decades. On the other hand, questions of political loyalty appeared relatively late in the dispute, and only after the mathematical terrain had been thoroughly worked over. Moreover, the key mathematical questions on which the dispute centered were not decided on the basis of social or political factors, but on straightforward mathematical grounds. Hobbes's numerous attempted quadratures and cube duplications won no adherents even among his friends and patrons, and the reason for this is evident: he was simply and spectacularly wrong. This is not to say that Hobbes failed in every aspect of his mathematics; after all (as we saw in chapter 4) he could pinpoint weaknesses in Wallis's mathematical work and his approach to questions in the philosophy of

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mathematics has some important strengths. Nevertheless, in his claims to have squared the circle or to have solved other great problems, Hobbes's efforts consistently fell short of the mark. This is perhaps best shown by the fact that his many quadratures all yield different results and thus cannot even be reconciled with one another, to say nothing of reconciling them with the truth.

8.3 HOBBES AND THE BETRAYAL OF RIGHT REASON

The absurdities to which Hobbes committed himself in defense of his geometric claims make it tempting to see him as driven by blind and irrational forces unconnected to mathematics, and considerations of explanatory symmetry might well make it seem that the same must be true of his opponents. After all, if we seek to understand the controversy from the point of view of the participants, and if we grant that Hobbes, at least, saw some significant merit in his claims to have squared the circle, we seem compelled to find some standpoint from which Hobbes's mathematical writings make sense. Wittgensteinian sociologists of knowledge will urge that it is only from within a form of life that utterances or other actions can be regarded as meaningful, and then conclude that Hobbes must have been part of an alternative (and apparently very exclusive) form of mathematical life, whose conflict with the prevailing norms must be examined symmetrically, i.e., without recourse to concepts like truth or error.

I think, however, that Hobbes himself outlines a better account of the matter. He possessed a keen psychological insight, particularly in matters of human motivation, and what he says about mankind can be taken to apply to his own case. In *Leviathan* he famously claimed to "put for a generall inclination of all mankind, a perpetuall and restlesse desire of Power after power, that ceaseth onely in Death" (*L* 1.11, 47; *EW* 3:85–86), while he also described ambitious men as "setting themselves against reason, as oft as reason is against them" (*L* 1.11, 50; *EW* 3:91). Hobbes plainly intended for his mathematical work to establish his standing at the forefront of European mathematics, and his mathematical ambitions were nothing less than a restless desire for the kind of power that comes with the reputation as a great savant. To have been mocked and humiliated at the hands of Wallis was, as Hobbes himself confessed, almost more than he could bear. His failures eventually turned him against the mathematics he had once declared as the pinnacle of human reason. His refusal to yield ground was the product of shattered ambition and wounded pride, as

well as his sense that he had nothing further to lose if his geometry were to go down to defeat. In fact, when Hobbes described the role of reason in controversy in *Leviathan,* his words would unwittingly be quite appropriate to his conflict with Wallis:

> And as in Arithmetique, unpractised men must, and Professors themselves may often erre, and cast up false; so also in any other subject of Reasoning, the ablest, most attentive, and most practised men, may deceive themselves, and inferre false Conclusions; Not but that Reason it selfe is always Right Reason, as well as Arithmetique is a certain and infallible Art: But no one mans Reason, nor the Reason of any one number of men, makes the certaintie; no more than an account is therefore well cast up, because a great many men have unanimously approved it. And therefore, as when there is a controversy in an account, the parties must by their own accord, set up for right Reason, the Reason of some Arbitrator, or Judge, to whose sentence they will both stand, or their controviersie must either come to blowes, or be undecided, for want of a right Reason constituted by Nature; so is it in all debates of what kind soever: And when men that think themselves wiser than all others, clamor and demand right Reason for judge; yet seek no more, but that things should be determined, by no other mens reason but their own, it is as intolerable in the society of men, as it is in play after trump is turned, to use for trump on every occasion, that suite whereof they have most in their hand. For they do nothing els, that will have every of their passions, as it comes to bear and sway them, to be taken for right Reason, and that in their own controversies: bewraying their want of right reason, by the claym they lay to it. (*L* 1.5, 18–19; *EW* 3:30–31)

There is probably no better description of Hobbes's own predicament. Convinced that he had delivered principles that could make short work of any mathematical problem, blinded by his passionate desire to defeat Wallis and reap his share of mathematical glory, and ultimately embittered by his failures, Hobbes himself betrayed his want of right reason by his claim to it.

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APPENDIX

Selections from Hobbes's Mathematical Writings

Hobbes's mathematical writings are a varied and voluminous lot produced over the course of more than three decades. The following selections are intended to provide a relatively detailed account of Hobbes's mathematics that can illustrate some of the points made in the course of the book. There can be no question of attempting a complete treatment of all Hobbes's mathematical efforts, not least because the frequency of technical errors and outright blunders make it an oeuvre that does not handsomely repay close attention. I have included the following mathematical items: Hobbes's contribution to Pell's 1647 refutation of Longomontanus; the 1656 and 1668 versions of part 3, chapter 17, article 2 of *De Corpore,* which present alternative demonstrations of the fundamental theorem for the quadrature of "deficient figures"; the original circle quadrature intended for part 3, chapter 20, article 1 of *De Corpore* (but not actually published) and the first of three quadratures printed in the twentieth chapter of the 1656 version of *De Corpore;* the comparison of the Archimedean spiral and the parabola from part 3, chapter 20, article 5 of *De Corpore;* the 1661 cube duplication published anonymously in Paris; and the first proposition from Hobbes's 1669 *Quadratura Circuli, Cubatio Sphaerae, Duplicatio Cubi breviter demonstrata,* which purports to square the circle.

A.1 HOBBES'S CONTRIBUTION TO PELL'S REFUTATION OF LONGOMONTANUS

This theorem was originally published in John Pell's *Controversiae de verâ circuli mensurâ . . . Prima pars* (Pell 1647, 50–51) as one of several alternative proofs of the result that can be stated as follows: if α is the tangent to an arc less than a quadrant of a circle, and β is the tangent to one-half of the same arc, and the circle has radius r, then $\alpha{:}\beta :: 2r^2{:}(r^2 - \beta^2)$. The primary interest in Hobbes's proof is that it shows his commitment to the classical theory of proportions. Where others who contributed to Pell's campaign against Longomontanus used more compressed "analytic" techniques and employed re-

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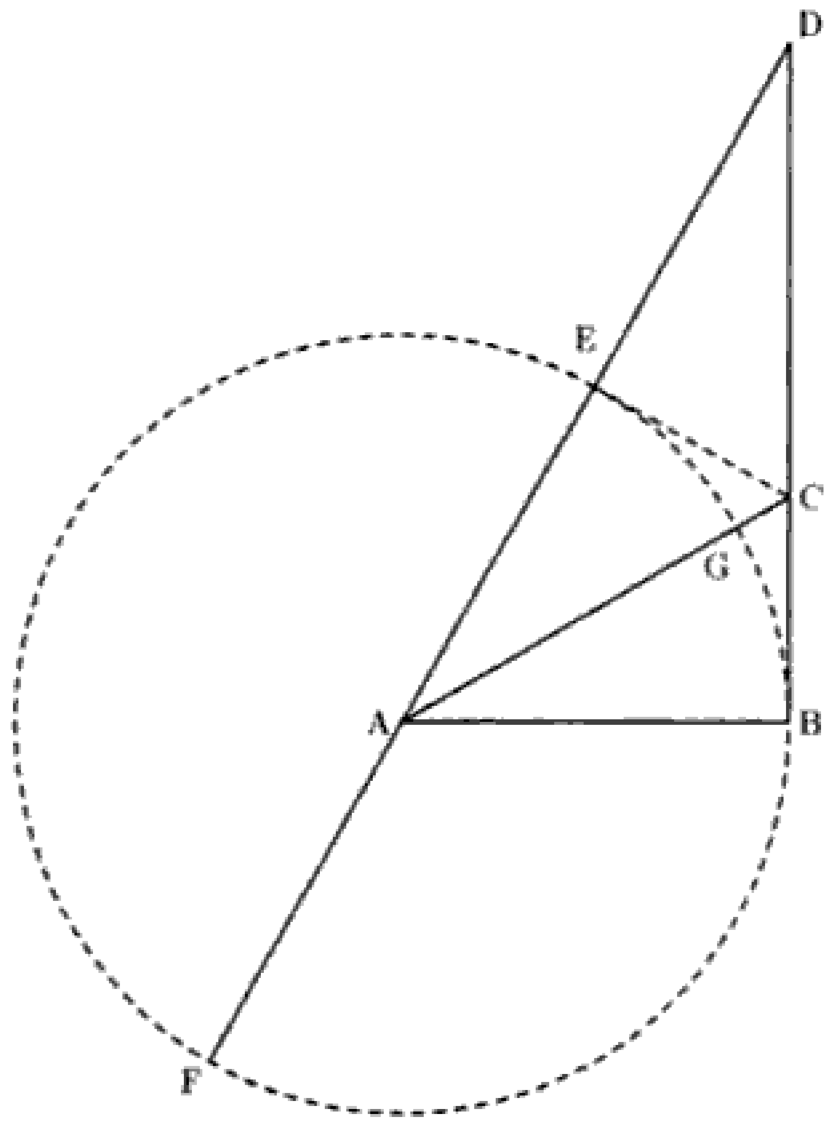

Figure A.1

latively short algebraic arguments, Hobbes's proof employs a rather prolix classical style characteristic of the "synthetic" demonstrations in Euclid's *Elements;* this is especially apparent in Hobbes's use of the various techniques for manipulating ratios and proportions, which he takes from the fifth book of Euclid. A manuscript version of Hobbes's proof survives as British Library MS. Add. 4278, f. 200ʳ, from which it is possible to date the work to June of 1645. I have made minor changes to Hobbes's notation (these include using 'AB^2' where he uses '*ABq*', expressing proportions by '*A:B* :: *C:D*' rather than Hobbes's preferred symbolism '*A.B* :: *C.D*', using the symbol '×' for multiplication, and adding parentheses where necessary to identify the factors in a product).

The tangent to an arc less than a quadrant is to the tangent of half that arc as twice the square of the radius to the square of the radius minus the square of the tangent of one half the arc.

Let *EBF* [in figure A.1] be a circle with center *A* and radius *AB*, and let the arc less than a quadrant *AB* be *BE*, half of which is *BG*. The tangent of the whole arc *BE* is the right line *BD*. The tangent of the arc *BG* is the right line *BC*.

I say $BD:BC :: 2AB^2:AB^2 - BC^2$.

Let the points *E, C* be connected. Then because the right lines *AB, AC* of the triangle *ABC* are equal to the right lines *AE, AC* of the triangle *AEC*, and

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the angle BAC is equal to the angle EAC, then by Euclid (*Elements* 1, prop. 4), the base EC is equal to the base BC, and the angle AEC is equal to the angle ABC, which is a right angle. Because the triangles DEC, DBA have equal angles at E and B, namely right angles, and they have the angle at D in common, they will be similar. Therefore the proportion $DB:AB :: DE:EC$ holds, and since EC and BC are equal, the proportion $DB:AB :: DE:BC$ also holds.[1] Permuting the terms of this proportion yields $DB:DE :: AB:BC$, and squaring the terms gives $DB^2:DE^2 :: AB^2:BC^2$. Now let DA be produced to F, and the rectangle contained by FD and DE (that is by $2AB + DE$ and DE) will be equal to DB^2. Thus DB is a mean proportional between $2AB + DE$ and DE. And therefore DB^2 is to DE^2 as $2AB + DE$ to DE. But it was shown that $DB^2:DE^2 :: AB^2:BC^2$. Therefore the proportion $AB^2:BC^2 :: 2AB + DE:DE$ holds. Further, by separating the ratio, the proportion $AB^2:AB^2 - BC^2 :: 2AB + DE:2AB$ is established.[2] And multiplying by the line DE, we have $AB^2:AB^2 - BC^2 :: (2AB + DE) \times DE:2(AB \times DE)$. But $(2AB + DE) \times DE$ is DB^2. Also, $2(AB \times DE)$ is equal to $2(DB \times BC)$ (since $DB:DE :: AB:BC$, as was shown above, and thus the rectangle contained by DB and BC is equal to the rectangle contained by AB and DE, and so $2(AB \times DE)$, the double of the second rectangle, is equal to $2(DB \times BC)$, the double of the first). Therefore $AB^2:AB^2 - BC^2 :: DB^2:2(DB \times BC)$ or $AB^2:AB^2 - BC^2 :: DB^2:2(BC - BD)$. And removing the common altitude DB, the proportion $AB^2:AB^2 - BC^2 :: DB:2BC$ arises. But because $2BC:BC :: 2AB^2:AB^2$, there will arise (*ex aequali* by a perturbed proportion, as in the scheme set out below),[3] the proportion $2AB^2:AB^2 - BC^2 :: DB:BC$. Which was to be demonstrated.

1. I use "the proportion $DB:AB :: DE:BC$ holds" and the like for Hobbes's rather awkward Latin "$DB.AB :: DE.BC$ sunt proportionales." At this point in the demonstration, Hobbes embarks on an argument based on the techniques for transforming ratios and proportions that are set out in book 5 of Euclid.

2. Euclid defines "separation of a ratio" in the fifteenth definition of book 5: "**Separation of a ratio** means taking the excess by which the antecedent exceeds the consequent in relation to the consequent itself." In other words, given the ratio A:B, the separation of the ratio is A − B:B (assuming that A is greater than B). The theorem that justifies the maneuver in Hobbes's demonstration is proposition 17 in book 5 of the *Elements:* "If magnitudes be proportional componendo, they will also be proportional seperando"; expressed symbolically, the theorem asserts that if A:B :: C:D, then (A − B):B :: (C − D):D.

3. In *Elements* (5, def. 17), Euclid declares "the ratio **ex æquali** arises when, there being several magnitudes and another set equal to them in multitude which taken two and two are in the same proportion, as the first is to the last among the first magnitudes, so is the first to the last among the second magnitudes; Or, in other words, it means taking the extreme terms by virtue of the intermediate terms." The eighteenth definition in book 5 reads: "A **perturbed proportion** arises when, there being three magnitudes and another set equal to them in multitude, as antecedent is to consequent among the first magnitudes, so is antecedent to consequent among the second magnitudes, while, as the

$$\begin{Bmatrix} 2AB^2 & & \\ AB^2 & \ldots\ \ldots & DB \\ AB^2 - BC^2 & \ldots\ \ldots & 2BC \\ & & BC \end{Bmatrix}$$

A.2 THE PRINCIPAL THEOREM IN THE QUADRATURE OF DEFICIENT FIGURES

The seventeenth chapter of *De Corpore* is devoted to the study of deficient figures produced by the motion of a line that diminishes as it moves through a given space. The determination of the area of such figures is the main goal of the chapter, and the key theorem asserts that if the ratio by which the line diminishes is in the *n*th power of the distance it has moved through the figure, then the ratio of the deficient figure to its complement is $1:n$. Expressed in more modern notation, this is a version of the result that $\int_0^a x^n dx = a^{n+1}/(n + 1)$. The remaining articles in chapter 17 of *De Corpore* contain a series of tables calculating the areas of various deficient figures and comparing them to one another. Wallis remarked that this chapter contains quite a few true propositions, notwithstanding the fact that the proof of the principal theorem is a failure. He concluded that, although he could not say where Hobbes had gotten them, "it is to be suspected that they are not yours, because they are true, while things of yours are wont to be false" (*Elenchus* 84).

Hobbes's presentation of this material has a very strong similarity to Cavalieri's method of indivisibles, most specifically in the fourth of his *Exercitationes Geometricae,* but it may also derive from Roberval's approach to the theory of indivisibles. One key point of similarity between Hobbes and the "indivisiblist" mathematicians is reflected in Hobbes's use of the language of indivisibles: he speaks of lines being drawn through "every possible part of a right line," refers to the latitude of small spaces as "indivisible," and describes a figure as "made up of so many indivisible spaces." The basic theorem is presented here in the version printed in the 1656 English translation of *De Corpore* and then in the version from the 1668 edition of *De Corpore.* The 1656 version contains two arguments for the result, the first of which was new to the English *De Corpore,* while the second is a reworked version of the

consequent is to a third among the first magnitudes, so is a third to the antecedent among the second magnitudes." Hobbes's use of these concepts is relatively straightforward: the previous argumentation has established two sequences of ratios that can be compared to one another, and the middle terms of both sequences can be removed by using the Euclidean definitions.

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argument in the original 1655 Latin *De Corpore.* The 1668 version gives a rather different version of the argument.

Although the various proofs differ in interesting ways, their similarities are also quite striking.[4] Hobbes attempts to establish his general result concerning the ratios of areas between a deficient figure and its complement, and in doing so he first attempts to establish that a ratio holds between pairs of lines in the deficient figure and its complement, after which he concludes that the same ratio will hold between the areas of the figure and its complement. This overall style of argument parallels the procedures of Cavalieri, but Hobbes's argumentation ultimately runs into serious difficulties in the details. The fact that the central result is given three distinct attempted proofs shows fairly clearly that Hobbes was made aware of the inadequacies in his argumentation, but (as inspection of the following material shows) the final version is hardly an improvement over those that preceded it. One important feature of Hobbes's argumentation is the fact that, although he speaks of figures as "made up" of indivisible elements, he does not attempt to represent the area of the figure as an infinite sum in the style of Roberval's *Traité des indivisibles* or Wallis's *Arithmetica Infinitorum.* Instead, he tries to work out a means of comparing the ratios of areas without using the sort of "algebraic" means he condemned as essentially ungeometrical.

A.2.1 De Corpore, *Part 3, Chapter 17, Article 2 (1656 Version)*[5]

A Deficient Figure, which is made by a Quantity continually decreasing to nothing by proportions every where proportionall and commensurable,[6] is to

4. Wallis took the differences between the Latin and English versions of chapter 17 of *De Corpore* as Hobbes's admission of error: "As to the Demonstration, you keep a vapouring (nothing to the purpose,) as if it were a good demonstration and not confuted. Yet, when you have done, (because you knew it to be naught) you leave it quite out in the English, and give us another (as bad) instead of it. That is, you confesse the charge. Your fundamentall Proposition was not demonstrated; and so this whole chapter comes to nothing" (*Due Correction* 119). This is a rather uncharitable reading of the evidence, since Hobbes did include the original argument (slightly modified in response to Wallis's criticisms) in the English version of *De Corpore.* Nevertheless, Wallis is correct in regarding Hobbes's argumentation as ultimately unconvincing.

5. I have corrected numerous typographical errors in the original and have made slight alterations to the diagram to make it fit the text. The frequency of typographical errors is particularly high in the second half of the article, which contains Hobbes's reworked version of the argument from the 1655 version *of De Corpore.* Almost all of these errors are in the labels for lines in the diagram. This suggests that in revising the argument to meet Wallis's criticisms Hobbes was either undecided about how best to proceed or making revisions hastily.

6. It should be remembered that Hobbes uses the English term *proportion* as a translation for the Latin *ratio*—a fact that leads to some significant linguistic confusions in

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its Complement, as the proportion of the whole altitude, to an altitude diminished in any time, is to the proportion of the whole Quantity which describes the Figure, to the same Quantity diminished in the same time.

Let the quantity *AB* [in figure A.2.1], by its motion through the altitude *AC*, describe the Complete Figure *AD;* and againe, let the same quantity, by decreasing continually to nothing in *C*, describe the Deficient Figure *ABEFC*, whose Complement will be the Figure *BDCFE*. Now let *AB* be supposed to be moved till it lie in *GK*, so that the altitude diminished be *GC*, and *AB* diminished be *GE;* and let the proportion of the whole altitude *AC* to the diminished altitude *GC*, be (for example) triplicate to the proportion of the whole quantity *AB* or *GK*, to the diminished quantity *GE*.[7] And in like manner, let *HI* be taken equal to *GE*, & let it be diminished to *HF;* and let the proportion of *GC* to *HC* be triplicate to that of *HI* to *HF;* & let the same be done in as many parts of the straight line *AC* as is possible; and a line be drawn through the points *B*, *E*, *F* and *C*. I say the deficient figure *ABEFC*, is to its Complement *BDCFE* as 3 to 1, or as the proportion of *AC* to *GC* is to the proportion of *AB*, that is, of *GK* to *GE*.

For (by the second Article of the 15. Chap.) the proportion of the complement *BEFCD* to the deficient figure *ABEFC* is all the proportions of *DB* to *OE*, and of *DB* to *QF*, and of all the lines parallel to *DB* terminated in the line *BEFC*, to all the parallels to *AB* terminated in the same points of the line *BEFC*.[8] And seeing the proportions of *DB* to *OE*, and of *DB* to *QF* &c. are

his account of the theory of ratios. Hobbes defines proportionality and commensurability of ratios at *DCo* 3.17.1; *EW* 1:247. The relevant definitions read as follows: "Four Proportions are said to be *Proportionall*, when the first of them is to the second, as the third is to the fourth. For example, if the first proportion be duplicate to the second; and again the third be duplicate to the fourth, those Proportions are said to be *Proportionall*. And Commensurable Proportions are those, which are to one another as number to number. As when to a proportion given, one proportion is duplicate, another triplicate, the duplicate proportion will be to the triplicate proportion as 2 to 3; but to the given proportion it will be as 2 to 1; and therefore I call those three proportions *Commensurable*." Despite some obscurity in the exposition, Hobbes's intent is fairly clear—when the ratio $\alpha:\beta$ is duplicate of $\gamma:\delta$, while the ratio $\varphi:\psi$ is likewise duplicate of $\chi:\omega$ then the pairs of ratios are proportional, which is essentially a generalization of the definition of four quantities standing in a proportion, i.e., in the same ratio. Similarly, commensurability is a matter of ratios being expressible in terms of integers.

7. The sixteenth article of chapter 13 of *De Corpore* (in the 1656 version) contains Hobbes's definition of multiplication of ratios: "A Proportion is said to be multiplied by a Number when it is so often taken as there be Unities in that Number" (*DeC* 2.13.16; *EW* 1:164). Thus, the ratio 1:3 taken twice is the ratio 1:9, or the ratio 5:4 taken twice is the ratio of 25:16; and in general if the ratio $\alpha:\beta$ is multiplied by n the result is $\alpha^n:\beta^n$. Thus, Hobbes is here assuming that $AC:GC = AB^3:GE^3$.

8. Hobbes here refers to a proposition added to the English version of chapter 15 that was not present in the 1655 Latin original. The proposition considers the velocities

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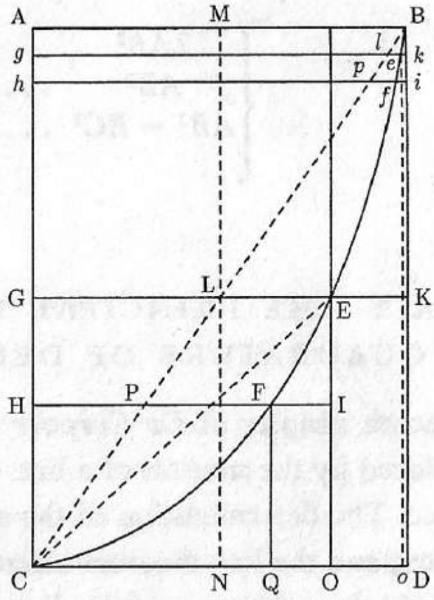

Figure A.2.1

every where triplicate of the proportions of *AB* to *GE*, and of *AB* to *HF* &c. the proportions of *HF* to *AB*, and of *GE* to *AB* &c. (by the 16 Article of the 13 Chap.)[9] are triplicate of the proportions of *QF* to *DB*, and of *OE* to *DB* &c. and therefore the deficient figure *ABEFC* which is the aggregate of all the lines *HF, GE, AB*, &c. is triple to the complement *BEFCD* made of all the lines *QF, OE, DB*, &c.; which was to be proved.[10]

It follows from hence, That the same complement *BEFCD* is 1/4 of the whole Parallelogram. And by the same method may be calculated in all other

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with which deficient figures are described, and it makes reference to the first figure of chapter 17 (figure A.2.1 here). The key portion of the proposition asserts that "as the proportions of the Swiftnesses wherewith *QF, OE, DB*, and all the rest supposed to be drawn parallel to *DB*, and terminated in the Line *BEFC*, are to the proportions of their several Times designed by the several parallels *HF, GE, AB* and all the rest supposed to be drawn parallel to the Line of time *CD*, and terminated in the Line *BEFC* (the aggregate to the aggregate) so is the Area or Plain *DBEFC* to the Area or Plain *ACFEB*" (*DCo* 3.15.2; *EW* 1:208–9). The point of the proposition is that throughout the curve *BEFC* the ratio between the parallels in the deficient figure to the parallels in its complement is constant, and indeed equal to the ratio of the areas of the deficient figure to its complement.

9. As mentioned above, this article is Hobbes's definition of multiplication of ratios.

10. The conclusion fails to follow, since from the fact that the lines are in triplicate proportion (i.e., in the ratio of cubes), nothing Hobbes has said thus far establishes that the areas are in the ratio of one to three. As Wallis remarked "this is but the same Bull that hath been baited so often. viz. *because the diameters* (DB, OE, QF, &c. that is CA, CG, CH,) *are in the triplicate proportion of the Ordinates* (AB, GE, HF,) *therefore the ordinates are in the triplicate proportion of the diameters*. . . . But how doe you prove this consequence? Nay, not a word of proof. We must take your word for it" (*Due Correction* 120).

Deficient Figures generated as above declared, the proportion of the Parallelogram to either of its parts; as that when the parallels encrease from a point in the same proportion, the Parallelogram will be divided into two equal Triangles; when one encrease is double to the other, it will be divided into a Semiparabola and its Complement, or into 2 and 1.

The same construction standing, the same conclusion may otherwise be demonstrated, thus.[11]

Let the straight line *CB* be drawn cutting *GK* in *L*, & through *L* let *MN* be drawn parallel to the straight line *AC*; wherefore the Parallelograms *GM* and *LD* will be equal. Then let *LK* be divided into three equal parts, so that it may be to one of those parts in the same proportion which the proportion of *AC* to *GC* or of *GK* to *GL* hath to the proportion of *GK* to *GE*. Therefore *LK* will be to one of those three parts as the Arithmetical proportion between *GK* and *GL* is to the Arithmetical proportion between *GK* and the same *GK* want the third part of *LK;* and *KE* will be somewhat greater then a third of *LK*.[12] Seeing now the altitude *AG* or *ML* is by reason of the continual decrease, to be supposed less than any quantity that can be given; *LK* (which is intercepted between the Diagonal *BC* and the side *BD*) will also be less then any quantity that can be given; and consequently, if *G* be put so neer to *A* in *g*, as that the difference between *Cg* and *CA* be less then any quantity that can be assigned, the difference also between *Cl* (removing *L* to *l*) and *CB*, will be less then any quantity that can be assigned; and the line *gl* being drawn & produced to the line *BD* in *k* cutting the crooked line in *e*, the proportion of *AC* to *gC* will still be triplicate to the proportion of *gk* to *ge*, and the difference between *k* and *e* and the third part of *kl* will be less then any quantity that can be given; and therefore the Parallelogram *eD* will differ from a third part of the Parallelogram *Ae* by a less difference then any quantity that can be assigned. Again, let *HI* be drawn parallel and equal to *GE*, cutting *CB* in *P*, the crooked line in *F*, and *EO* in *I*, and the proportion of *Cg*, to *CH* will be triplicate to the proportion of *ge* to *HF*, and *IF* will be greater then the third part of *PI*. But again,

11. What follows is a reworked version of the argument in the original 1655 *De Corpore*.

12. In the original version of this argument, Hobbes claimed that *KE* would be one-third of *LK* (Hobbes 1655, 145). Wallis objected that, since the point *G* is taken arbitrarily, the ratio between *KE* and *LK* could take on any desired value and the argument therefore fails (*Elenchus* 70). In the *Six Lessons*, Hobbes replied that "you did not then observe, that I make *the Altitude AG, less then any Quantity given,* and by consequence *EK* to differ from a third part by a less difference then any Quantity that can be given" (*SL* 5; *EW* 7:300). Hobbes therefore modified the argument by admitting that the ratio differs from the desired result, but that the difference can be made as small as desired.

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setting *H* in *h* so neer to *g*, as that the difference between *Ch* and *Cg* may be as but a point, the point *P* will also in *p* be so neer to *l*, as the difference between *Cp* and *Cl* will be but as a point; and drawing *hp* till it meet with *BD* in *i*, cutting the crooked line in *f*, and having drawn *eo* parallel to *BD*, cutting *DC* in *o*, the Parallelogram *fo* will differ less from the third part of the Parallelogram *gf*, then by any quantity that can be given. And so it will be in all other Spaces generated in the same manner. Wherefore the differences of the Arithmetical and Geometrical Means,[13] which are but as so many points *B*, *e*, *f*. &c. (seeing the whole Figure is made up of so many indivisible Spaces) will constitute a certain line, such as the line *BEFC*, which will divide the complete Figure *AD* into two parts, whereof one, namely *ABEFC*, which I call a Deficient Figure, is triple to the other, namely *BDCFE*, which I call the Complement thereof, And whereas the proportion of the altitudes to one another, is in this case everywhere triplicate to that of the decreasing quantities to one another; in the same manner if the proportion of the altitudes had been every where quadruplicate to that of the decreasing quantities it might have been demonstrated, that the Deficient Figure had been quadruple to its Complement; and so in any other proportion, Wherefore, a Deficient Figure, which is made, &c. Which was to be demonstrated.

A.2.2 De Corpore, *Part 3, Chapter 17, Article 2 (1668 Version)*

A deficient figure made by a quantity continually decreasing by ratios everywhere proportional and commensurable until it evanesces is to its complement as the ratio of the whole altitude to an altitude diminished in any time is to ratio of the whole quantity which describes the figure to the same quantity diminished in the same time.

Let the parallelogram *ABCD* (figure A.2.2) be described, and let the base *AB* be understood to be moved parallel to *CD*, so that as it is moved it perpetually decreases until it evanesces in point *C*; and let the ratio of *AB* diminished to the same whole *AB* be everywhere the same as the ratio of *AC* to *AG*, either everywhere duplicate, or triplicate, or in any other ratio of a ratio to a ratio. While *AB* decreases in this way the point *B* describes a certain line, say *BEFC*. Now I say that if the ratio of *AC* to *AG* is the same as the ratio of *AB* to *GE*,

13. Hobbes claims in the corollary to *DCo* 2.13.28 that "if any quantity be supposed to be divided into equal parts infinite in number, the difference between the Arithmetical and Geometrical Means will be infinitely little, that is, none at all; and upon this foundation chiefly, the Art of making those Numbers which are called *Logarithmes* seems to have been built." This principle was also invoked in the 1655 version of the argument. It is unclear to me just how Hobbes thinks that this principle can assist in his argument.

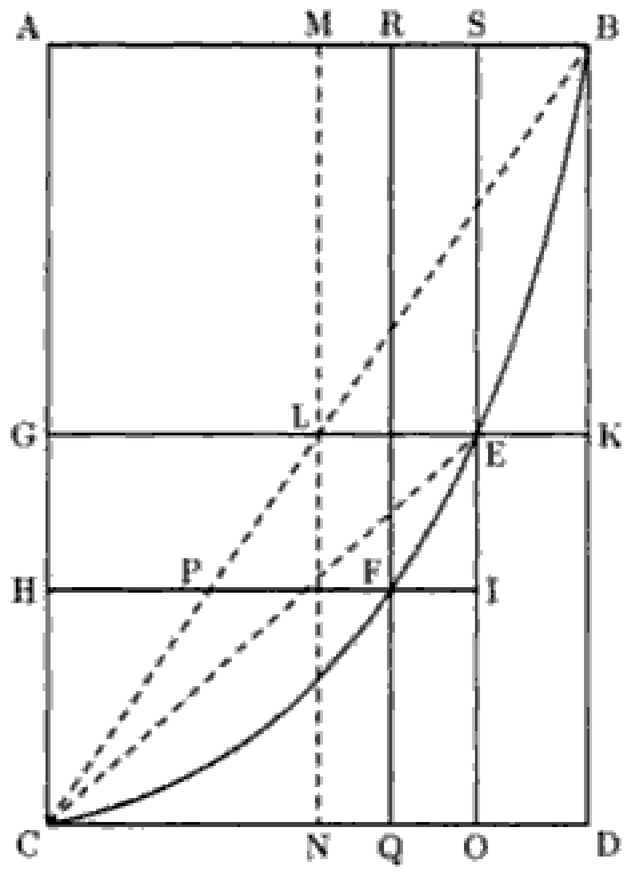

Figure A.2.2

then the space *ABEFC* is to the space *DCFEB* as one to one; but if the ratio of *AC* to *AG* is the duplicate of the ratio of *AB* to *GE*, then the space *ABEFC* is to the space *DBEFC* as two to one; if triplicate, as three to one, and so on.

LEMMA 1

By whatever ratio the velocity of a moved point is increased, in the same ratio are also increased the spaces passed over by the point in the same or equal times.

LEMMA 2

If between two right lines there are interposed an infinite number of both arithmetical and geometrical means, these do not differ in magnitude.[14]

In the parallelogram *ABCD* let the side *AB* be understood to be moved parallel to the side *CD*, and in moving to decrease until at last it evanesces in the point *C*; and by such motion the figure *ABEFC* is described, leaving the complement whose line *BEFC* is described by the endpoint *B* of the decreasing line *AB*. And in the same time let the side *AC* be understood to move uniformly to *BD*. Thus *CD* can be taken for the measure of time. But the right lines parallel to the line *CD*, terminated on one side in the line *BEFC*, and on the other in the right line *AC*, will be measures of the parts of time in which *AB* is moved to *CD* and *AC* to *BD*.

14. This is just the corollary to *DCo* 2.13.28 that Hobbes appealed to in his original argument, as well as in the revised version of that argument from 1656. The first lemma recapitulates principles from *DCo* 3.16, articles 1–6.

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Now let there be taken arbitrarily in the right line *CD* a point *O*, and let *OS* be drawn parallel to the side *BD*, cutting the line *BEFC* in *E* and the right line *AB* in *S*. Again, from the point *Q* taken arbitrarily in *CD* let *QR* be drawn parallel to the same side *BD*, cutting *BEFC* in *F* and *AB* in *R*. And let *EG, FH* also be drawn, parallel to *CD* and cutting *AC* in *G* and *H*. Finally, let the same be supposed to be done in every point of the line *BEFC*.

I say that as the aggregate of all the velocities by which the right lines *QF, OE, DB*, and all the rest generated in the same manner is to the aggregate of the times designated by the right lines *HF, GE, AB*, and the rest, so the plane surface *DCFEB* is to the plane surface *ABEFC*. Just as *AB* by decreasing through the line *BEFC* in the time *CD* evanesces in point *C*, so *CD* (equal to *AB* itself) by decreasing through the same line *CFEB* in the same time evanesces in the point *B*, having described the right line *DB* equal to *AC*. Therefore the velocities with which *AC* and *DB* are described are equal to one another. On the other hand, seeing that in the same time in which the point *O* describes the right line *OE* the point *S* describes the right line *SE*, then *OE* will be to *SE* as the velocity with which *OE* is described to the velocity with which *SE* is described. And for the same reason *QF* will be to *RF* as the velocity with which *QF* is described to the velocity with which *RF* is described, and thus for all the other parallels. Therefore as the right lines that are parallel to the side *AB* and terminated in the line *BEFC* are the measures of the times, so the right lines that are parallel to the side *BD* (and terminated in the same line *BEFC*) are the measures of the velocities. Now (by lemma 1) in whatever ratio the velocities are increased, in the same ratio the right lines passed over in the same times are increased, namely *QF, OE, DB*, etc.

Now all the lines *QF, OE, DB*, etc. constitute the plane surface *DBEFC*; and all the lines *HF, GE, AB*, etc.—that is, all of *ES, FR, CA*, etc.—constitute the plane surface *ACFEB*. The former of these are the aggregate of the velocities, the latter the aggregate of times. Thus as the aggregate of the velocities is to the aggregate of the times, so the complement *DBEFC* is to the figure *AB-EFC*. Therefore if indeed the ratios of *DB* to *OE* and *OE* to *QF* should be (for example) triplicate, then vice versa the ratios of *OE* to *DB*, and *QF* to *OE* will be subtriplicate of the ratios of *GE* to *AB* and *HF* to *GE*. Thus the aggregate of all of *QF, OE, BD*, etc. will be to the aggregate of all *HF, GE, AB*, etc. (by lemma 2) subtriple.[15] Therefore as the aggregate of the velocities is to the aggregate of the times by which the deficient figure is described, so will the complement of the figure to the deficient figure itself, that is to say the complement *DBEFC* to the figure *ABEFC*. Which was to be demonstrated.

15. Even the most charitable reading of Hobbes's principle leaves it obscure why this consequence should follow.

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A.3 TWO OF HOBBES'S QUADRATURES FROM *DE CORPORE*, PART 3, CHAPTER 20

I here present two versions of the several attempted quadratures that Hobbes at various times intended for chapter 20 of *De Corpore*. The first is the original effort that Hobbes planned to include, but he abandoned when *De Corpore* was in proofs. Wallis acquired a copy of this from Hobbes's printer and quoted from it at great length (*Elenchus* 97–107). I have reconstructed the proof from Wallis's quotation of the unpublished sheets. The second is the first of three attempted quadratures published in the 1656 English version of *De Corpore*. Its construction is the same as that of the unpublished first quadrature, but Hobbes no longer claims it as an exact result. Instead, he declares that in his twentieth chapter "I have let stand there that which I did before condemn, not that I think it exact, but partly because the Division of Angles may be more exactly performed by it then by any organicall way whatsoever" (*SL* epistle; *EW* 7:186). Hobbes's indecision and frustration are palpable in the second half of the 1656 version of the argument: he grants that his construction conflicts with established values for π, but he cannot quite see the flaw in his procedures and is unwilling to let the matter rest. I find these two demonstrations more interesting than many other Hobbesian attempts at quadrature because they give a better picture of the "method of motions" he used to approach the problem and from which he expected such great results. This method attempts to rectify curvilinear arcs by imagining them to be straightened as points on the arc move uniformly through specified points in the construction. As is so often the case with Hobbes's mathematics, his constructions in both cases are quite complex (with a number of otiose lines cluttering the diagram) and the argumentation ultimately fails to achieve the desired result. Nevertheless, it is instructive to see just what sort of argument Hobbes had first found persuasive as he prepared chapter 20 of *De Corpore*.

A.3.1 The Original Quadrature Intended for De Corpore, *Part 3, Chapter 20, Article 1*

To find a right line equal to the perimeter of a circle.

Let *ABD* [in figure A.3.1] be the quadrant of a circle, about which is circumscribed the square *ABCD*. Let the sides of the square be bisected at *E*, *F*, *G*, and *H*. Then the lines *FH*, *EG* will divide $\widehat{BD}$ into three equal parts at *I* and *K*. Let *IM* (the sine of $\widehat{BI}$) be drawn, and *IM* will be half of the radius *BC*. Let $\widehat{BI}$ be divided into four equal parts at *L*, *N*, and *O*, and let the right line *MI* be so divided at *P*, *Q*, and *R*. Let *PL*, *QN*, and *RO* be connected, and to *SN* (the sine of $\widehat{BN}$) let *NT* be added, which is equal to *SN* itself. Finally, let *IT* be drawn and produced to meet *BC* in *V*. I say the right line *BV* is equal to

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$\widehat{BI}$, and thus three times BV (which is Be) is equal to the arc BD, and twelve times the same BV is equal to the perimeter of the circle of which ABD is a quadrant.[16]

Because both MQ and QI, as well as SN and NT, are equal, then as ST is to SN, so MI is to QI. Let the right line TI be produced to meet BA produced in q. Then NQ produced falls on the same point q. And qP, qQ, qR joined and produced to BV will cut BV in four equal parts at X, Y, and Z.

On the right lines MQ, QI let two equilateral triangles MgQ, QhI be constructed, and with centers g and h let $\widehat{MQ}$, $\widehat{QI}$ be drawn. Either of these is equal to either of $\widehat{BN}$, $\widehat{NI}$.[17] Again, on the right lines MP, PQ, QR, RI let there be constructed as many equilateral triangles MkP, PlQ, QpR, RbI. And with centers k, l, p, b let $\widehat{MP}$, $\widehat{PQ}$, $\widehat{QR}$, $\widehat{RI}$ be drawn, any one of which is equal to any of $\widehat{BL}$, $\widehat{LN}$, $\widehat{NO}$, $\widehat{OI}$.

Now because $\widehat{IQ}$, $\widehat{IN}$ are equal, rectilinear motion through qQ places the point Q in N; and $\widehat{IN}$, $\widehat{IQ}$ will coincide, of course $\widehat{IQ}$ (which is slightly more curved than $\widehat{IN}$) being slightly straightened. And because $\widehat{IR}$, $\widehat{RQ}$ are both together equal to $\widehat{IN}$, these are straightened by the same motion and are placed in $\widehat{IN}$, with which they coincide. And thus by the rectilinear motion through qR the midpoint R, which is brought to the middle of the right line YV, will be brought to the middle of $\widehat{IN}$, that is through O.[18]

Similarly, because motion through qQ places the point Q in N, and motion through qM places the point M in B, and $\widehat{MQ}$, $\widehat{BN}$ are equal, $\widehat{MQ}$ coincides with $\widehat{BN}$. And by the same motion $\widehat{MP}$, $\widehat{PQ}$ will be placed in the same $\widehat{BN}$, with which they coincide. Therefore, the motion through qP places the midpoint P in the middle of $\widehat{BN}$, that is in L.[19]

In the same manner, by the perpetual bisection of the right line MI, and by constructing equilateral triangles on the parts thus produced, there will arise an infinity of arcs, that is, as many as one might wish, equal to each other

16. Elementary trigonemetric calculation shows that this assertion is false. By construction (taking the radius AB as a unit), $ST = 2\sin(15°)$, $MI = 1/2$, and $SM = \cos(15°) - \cos(30°)$. BV therefore has a length of approximately .5236539, and Hobbes requires a value for π at aproximately 3.1419234.

17. More precisely, taking AD as unit, $\widehat{BN} = \widehat{NI} = \pi/12 = \widehat{MQ} = \widehat{QI}$.

18. Wallis observes that this is the crucial misstep in the argument: "In no way [will the point R pass through O], for the right line qRZ does not pass through O. Although the points q, Q, N lie in the same right line, as also do q, M, B, nevertheless q, R, O do not, nor do q, P, L lie in the same line. And if you should contend that they do, it remains for you to prove it" (*Elenchus* 98). From here forward, the argument proceeds from a false supposition and delivers the inevitable false result.

19. Hobbes's "method of motions" fails here again. He has no guarantee that qP continued will pass through L, and calculation shows that, indeed, it does not. In effect, he is assuming what he needs to prove, namely that the fourth part of the line BV is equal to a fourth part of $\widehat{BI}$.

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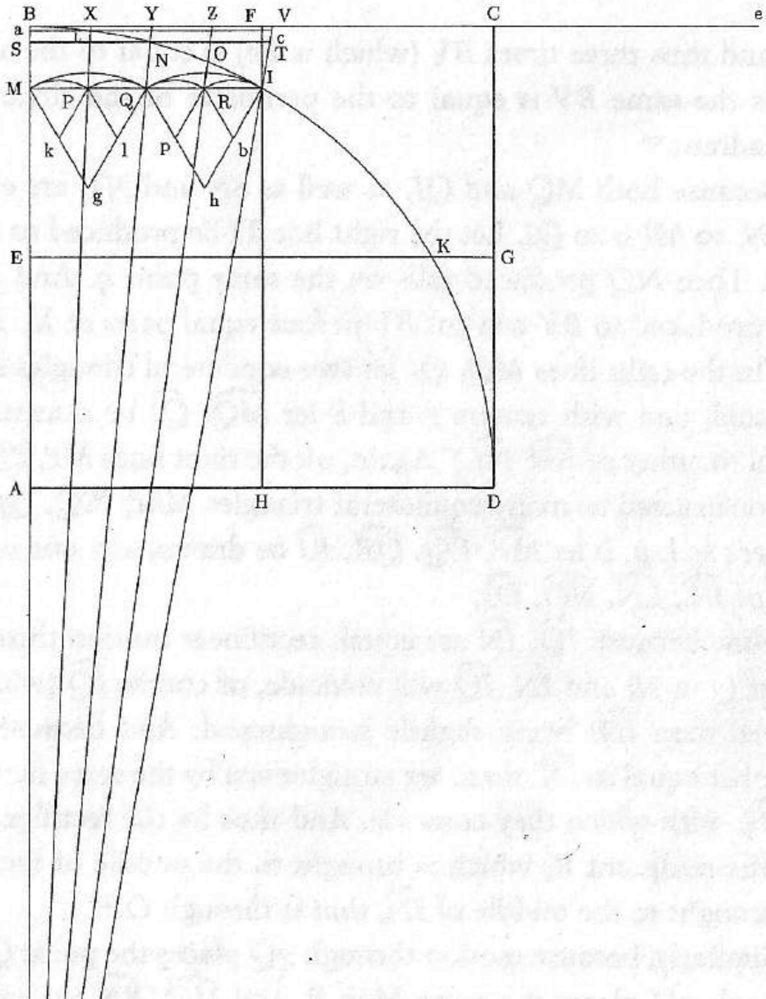

Figure A.3.1

and all taken together equal to $\widehat{BI}$. And by motion through the right lines drawn from q through the several parts of division of the right line MI, these arcs will be placed in as many equal parts of the right line BV. But the same right lines drawn from q will cut the right lines BV and MI, and $\widehat{BI}$ in the same ratios.[20]

20. Had the previous argumentation succeded, the demonstration could have ended here because the determination of the arbitrary arc lengths for the arc BI would suffice

Let *aL* (the sine of $\widehat{BL}$) be drawn, and let it be produced until it cuts *IV* at *c*. Then, because *MP* is the fourth part of *MI*, *aL* will be the fourth part of *ac*. And because *qB* is greater than *qa*, *BV* will also be greater than *ac*. Therefore *BV* is greater than four sines of $\widehat{BL}$, which arc is one fourth of $\widehat{BI}$. In the same way it can be shown that if $\widehat{BI}$ were divided into any number of equal parts (so that the difference between the arc itself and the aggregate of as many sines of one of these smallest parts as there are parts in the division is less than any given quantity), the right line *BV* would still be greater than all of these sines taken together. Therefore the right line *BV* is not less than the arc *BI*. But it cannot be greater either, because if this point *B* itself is taken for the sine of the smallest part of the arc, then the aggregate of all the sines considered as points is the right line *BV* itself, and it is equal to the arc *BI*.[21] Therefore the right line *BV* is equal to the arc *BI*, and *Be* is equal to the arc *BD*, and four times *Be* is equal to the perimeter of the circle of which *ABD* is a quadrant. Therefore a right line has been found equal to the perimeter of the circle, which was to be done.

A.3.2 The First Quadrature from the 1656 De Corpore

Let the Square *ABCD* [in figure A.3.2] be described, and with the *Radii AB*, *BC* and *DC* the three Arches *BD*, *CA* and *AC*; of which let the two *BD* and *CA* cut one another in *E*, and the two *BD* and *AC* in *F*. The Diagonals therefore *BD* and *AC* being drawn will cut one another in the center of the Square *G*, and the two Arches *BD* and *CA* into two equal parts in *H* and *Y*; and the Arch *BHD* will be trisected in *F* and *E*. Through the Center *G* let the two Straight Lines *KGL* and *MGN* be drawn parallel and equal to the sides of the Square *AB* and *AD*, cutting the four sides of the same Square in the points *K*, *L*, *M* and *N*; which being done, *KL* will pass through *F*, and *MN* through *E*. Then let *OP* be drawn parallel and equal to the side *BC*, cutting the Arch *BFD* in *F*, and the sides *AB* and *DC* in *O* and *P*. Therefore *OF* will be the sine of the arch *BF*, which is an arch of 30 degrees; and the same *OF* will be equal to half the Radius. Lastly, dividing the arch *BF* in the middle in *Q*, let *RQ* the Sine of the

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for the quadrature of the circle. Hobbes continues, however, and the remaining argumentation resembles the kind of double reductio ad absurdum argument found in Archimedean exhaustion proofs. His idea is to show that the line *BV* can be neither greater nor less than the arc *BI*. Unfortunately for Hobbes, his argument begs the crucial question.

21. This part of the argument begs the question, as Hobbes later realized. In the printed version of the 1655 *De Corpore* he admitted, "Seeing that it is possible that the line *qP* produced to the perpendicular *Li* may cut it beyond *L*, it is also possible that *aL* is greater than one-fourth of the right line *ac*. Therefore it is not demonstrated that the right line *BV* is equal to the arc *BI*" (Hobbes 1655, 171).

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